~~P-75~~ Curvature and torsion of a curve given as the intersection of two surfaces.

Proof:⇒ Let $r = r(s)$ —— (1)

be the curve of intersection of two surface

$f(r)=0, \quad \phi(r)=0$ —— (2)

The vector normal to surface (2)

$$\left.\begin{aligned}\nabla f &= \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)\\ \text{And} \quad \nabla \phi &= \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right)\end{aligned}\right\} \longrightarrow (3)$$

Let t is the tangent vector then

$\nabla f \times \nabla \phi \parallel t$

∇f, $\nabla \phi$ is normal two normal Product will gone $\parallel$ to tangent

$\Rightarrow \nabla f \times \nabla \phi = \lambda t = \lambda r'$ —— (4)

λ is proportional constant

Let a set $\nabla f \times \nabla \phi = h = (h_1, h_2, h_3)$

$\lambda t = \lambda r' = h$ —— (5)

square $\lambda^2 = h^2$ —— (6) $\quad \boxed{t \cdot t = 1}$

$\lambda r' = h \Rightarrow \lambda(x', y', z')$

$= (h_1, h_2, h_3)$

$r = (x, y, z)$

$r' = (x', y', z')$

Compare

$\Rightarrow \lambda x' = h_1, \; \lambda y' = h_2, \; \lambda z' = h_3$

$\lambda \frac{dr}{ds} = \lambda\left(\frac{\partial r}{\partial x}\frac{dx}{ds} + \frac{\partial r}{\partial y}\frac{dy}{ds} + \frac{\partial r}{\partial z}\frac{dz}{ds}\right)$

$\lambda \frac{dr}{ds} = \lambda x' \frac{\partial r}{\partial x} + \lambda y' \frac{\partial r}{\partial y} + \lambda z' \frac{\partial r}{\partial z}$ $\longleftarrow$ $\boxed{\frac{dx}{ds} = x'}$

$\lambda \frac{dr}{ds} = \left(h_1 \frac{\partial}{\partial x} + h_2 \frac{\partial}{\partial y} + h_3 \frac{\partial}{\partial z}\right) r$ $\longleftarrow$ $\lambda x' = h_1$, $\lambda y' = h_2$, $\lambda z' = h_3$

$= \Delta$ (Let) —— (H)

$r = (x, y, z)$

$\frac{dr}{ds} = \frac{\partial r}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial r}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial r}{\partial z}\frac{\partial z}{\partial s}$

$\lambda \frac{dr}{ds} = h$ From (5)

$\Delta r = h$ —— (7)

But $\lambda t = h$ From (5)

Test

01/5/19

~~Note:- All Questions,~~

(1) Serret- Frenet Formulae.

(2) A necessary and sufficient condition for a curve to be a straight line is that the curvature $K=0$ at all points of the curve.

(3) Curve is helix $\Leftrightarrow \frac{K}{\tau}$ = Constant.

—x—

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